Fluctuation Noise in Vacuum Tubes *

By G. L. PEARSON

The fluctuation noises originating in vacuum tubes are treated theoretically under the following headings: (1) *thermal agitation* in the internal plate resistance of the tube, (2) *shot effect and flicker effect* from space current in the presence of space charge, (3) *shot effect* from electrons produced by collision ionization and secondary emission, and (4) *space charge fluctuations* due to positive ions. It is shown that thermal agitation in the plate circuit is the most important factor and should fix the noise level in low noise vacuum tubes; shot noise and flicker noise are very small in tubes where complete temperature saturation is approached; shot noise from secondary electrons is negligible under ordinary conditions; and noise from space charge fluctuation due to positive ions is usually responsible for the difference between thermal noise in the plate circuit and total tube noise.

A method is deduced for the *accurate rating of the noise level of tubes* in terms of the input resistance which produces the equivalent thermal noise. *Quantitative noise measurements* by this method are reported on four different types of vacuum tubes which are suitable for use in the initial stage of high gain amplifiers. Under proper operating conditions the noise of these tubes approaches that of thermal agitation in their plate circuits at the higher frequencies and is 0.54 to 2.18×10^{-18} mean square volts per cycle band width in the frequency range from 200 to 15,000 cycles per second. Below 200 cycles per second the noise is somewhat larger.

The minimum noise in different types of vacuum tube circuits is discussed. These include input circuits for high gain amplifiers, ionization chamber and linear amplifier for detecting corpuscular or electromagnetic radiation, and photoelectric cell and linear amplifier for measuring light signals.

With the aid of these results it is possible to design circuits having the maximum signal-to-noise ratio obtainable with the best vacuum tubes now available.

INTRODUCTION

IT is well known that the noise inherent in the first stage of a high gain amplifier is a barrier to the amplification of indefinitely small signals. Even when fluctuations in battery voltages, induction, microphonic effects, poor insulation, and other obvious causes are entirely eliminated, there are two sources of noise which remain, namely, thermal agitation of electricity in the circuits and voltage fluctuations arising from conditions within the vacuum tubes of the amplifier. The effect of thermal agitation in circuits outside the vacuum tube is well understood, but in the case of tube noise there is considerable confusion. In order to clarify the whole subject, the present paper analyzes the various sources of noise in vacuum tubes and their attached circuits, points out a new method for the measurement of tube noise, reports the results of such measurements on four different types of vacuum tubes, and discusses the minimum noise in different types of vacuum tube circuits.

Often, in the use of high-gain amplifiers, the impedance of the input circuit is naturally high or may effectively be made high by the use of a transformer. In this case the contribution of noise from the vacuum tube is small compared with the noise arising from thermal agitation in the input circuit. This is a desirable condition since it furnishes the largest ratio of signal to noise for a given input power. Sometimes, however, the input impedance is perforce so small that the tube noise may be comparable with or greater than the thermal agitation noise. Such conditions may arise, for example, in amplifiers where the frequency dealt with is high or the frequency range is wide. It is, therefore, desirable to know the noise level to be expected from different types of tubes that may be used in the first stage of high-gain amplifiers as well as to be able to calculate the thermal noise level of the input circuit.

The noise of thermal agitation [1] arises from the fact that the electric charge in a metallic conductor shares the thermal agitation of the molecules of the substance so that minute variations of potential difference are produced between the terminals of the conductor. The mean square potential fluctuation is proportional to the absolute temperature and to the resistive component of the impedance of the conductor, but is independent of the material. The thermal noise power is distributed equally over all frequencies although the apparent magnitude depends on the electrical characteristics of the measuring system as well as on those of the conductor itself. From purely theoretical considerations the following equation has been derived [2] to give the thermal noise voltage at the output of an amplifier due to the thermal agitation of electric charge in an impedance at the input:

$$\overline{E_T^2} = 4kT \int_F R(f)\,|G_1(f)|^2 df. \tag{1}$$

$\overline{E_T^2}$ is here the mean square thermal noise voltage across the measuring device, k is Boltzmann's constant (1.37×10^{-23} watt second per degree), T the temperature of the impedance expressed in degrees Kelvin, $R(f)$ the resistive component of the impedance at the frequency f, $G_1(f)$ the voltage amplification between the input impedance and the measuring device at the frequency f, and F the frequency band within which the amplification is appreciable.

While the thermal noise in the circuit is accurately predictable, the noise originating within the vacuum tube is not completely under-

[1] J. B. Johnson, *Phys. Rev.*, **32**, 97 (1928).
[2] H. Nyquist, *Phys. Rev.*, **32**, 110 (1928).

stood and cannot be calculated accurately. It is known, however, that tube noise arises from a number of different causes, chief among which are: (1) thermal agitation in the internal plate resistance of the tube, (2) shot effect and flicker effect from space current in the presence of space charge, (3) shot effect from electrons produced by collision ionization and secondary emission, and (4) space charge fluctuations due to positive ions. Each of these sources of noise will be discussed in the following section:

ORIGIN OF NOISE IN THERMIONIC AMPLIFIER TUBES

Thermal Agitation in the Internal Plate Resistance of the Tube [3]

Just as voltage fluctuations are produced by thermal agitation in resistances comprising the input circuit, so the resistance component of the impedance between plate and cathode is a source of thermal noise. This impedance consists of the internal plate impedance of the vacuum tube in parallel with the external load impedance. Llewellyn [4] has shown that the resistive component of the internal plate impedance produces thermal noise as if it were at the temperature of the cathode. The following formula has been developed by him to cover the case where the tube impedance and load impedance are pure resistances:

$$\overline{E_T{}^2} = 4k[(r_p r_0)/(r_p + r_0)^2](T_0 r_p + T_f r_0) \int_F |G_2(f)|^2 df. \qquad (2)$$

Here r_p is the internal plate resistance of the tube, r_0 the external load resistance in the plate circuit, $G_2(f)$ the voltage amplification between the load resistance r_0 and the measuring device, and T_0 and T_f respectively the temperatures of the external load resistance and cathode expressed in degrees Kelvin. The relationship between $G_1(f)$ and $G_2(f)$ is given by

$$G_1(f) = G_2(f)(\mu r_0)/(r_0 + r_p), \qquad (3)$$

where μ is the voltage amplification factor of the tube. By assuming $G_2(f)$, in equation (2), to be constant over the frequency range F and substituting for it the value given by equation (3), it is found on integrating that the thermal noise in the plate circuit of the tube produces the same effect in the measuring device as a signal applied

[3] During the preparation of this paper a paper by E. B. Moullin and H. D. Ellis entitled "Spontaneous Background Noise in Amplifiers Due to Thermal Agitation and Shot Effects" appeared in *I. E. E. Jour.*, **74**, 323 (1934). The authors there contend that no thermal noise is produced in the plate impedance of a thermionic vacuum tube and that shot noise is not altered by the presence of space charge. With these contentions I cannot agree and I hope to state my definite reasons therefor at a later date.

[4] F. B. Llewellyn, *Proc. I. R. E.*, **18**, 243 (1930).

to the input circuit whose magnitude at the grid expressed in mean square volts is given by

$$\overline{V^2} = 4kT_0(r_p/\mu)^2[T_f/(T_0 r_p) + 1/r_0]F. \tag{4}$$

Since the noise of thermal agitation is always present, this equation gives the absolute minimum to which fluctuation noise in an amplifying tube can be reduced after all other causes have been eliminated. It shows that for the ideal low noise tube in which thermal noise in the plate circuit is the limiting factor, the noise level may be reduced by a decrease in the cathode temperature, a decrease in the effective frequency band, or by an independent decrease in the plate resistance or increase in the amplification factor. In order to operate at a minimum noise level the tube should work into a load resistance which is large in comparison with $r_p T_0/T_F$. Under this circuit condition the noise level is inversely proportional to μ^2/r_p, a quantity often defined as the "figure of merit" of an amplifying tube.

Shot Effect and Flicker Effect in the Presence of Space Charge

The theory of the shot effect in the absence of space charge has been studied quite completely both theoretically and experimentally by many investigators.[5] The results, however, are not applicable to the study of noise in thermionic vacuum tubes used in high-gain amplifiers, since a high degree of space charge is required in tubes used for this purpose. Llewellyn has extended the theory of the shot effect to cases where partial temperature saturation exists, and obtained a general equation to cover all conditions.[4] This equation reduces to the following form when the load impedance is a pure resistance:

$$\overline{E_g^2} = 2ej(\partial i/\partial j)^2[r_p r_0/(r_p + r_0)]^2 \int_F |G_2(f)|^2 df. \tag{5}$$

$\overline{E_g^2}$ is here the mean square shot voltage across the measuring device, i the total space current, j the total current emitted by the cathode, and e the electronic charge (1.59×10^{-19} coulomb).

A precise experimental verification of this equation is very difficult because of the difficulty in determining $\partial i/\partial j$ accurately. Thatcher,[6] however, has made shot measurements in the presence of space charge ($1 \gtrless \partial i/\partial j \gtrless 0.66$) which verify the theory within the experimental error of the determination of $\partial i/\partial j$.

[5] W. Schottky, *Ann. d. Physik*, **57**, 541 (1918); T. C. Fry, *Jour. Franklin Inst.*, **199**, 203 (1925); A. W. Hull and N. H. Williams, *Phys. Rev.*, **25**, 147 (1925).
[6] Everett W. Thatcher, *Phys. Rev.*, **40**, 114 (1932).

Equation (5) shows that as long as space charge is too small to affect the flow of current, that is when i is equal to j, the mean square shot voltage is directly proportional to the space current. As emission is increased, however, space charge begins to control and finally limits the space current so that the value of $\partial i / \partial j$ approaches zero. Thus the shot voltage increases less rapidly as space charge becomes effective and then finally decreases rapidly toward zero as complete space charge control is reached.

Experimental curves showing the effect of space charge on tube noise are shown in Fig. 1 where abscissæ represent space current in
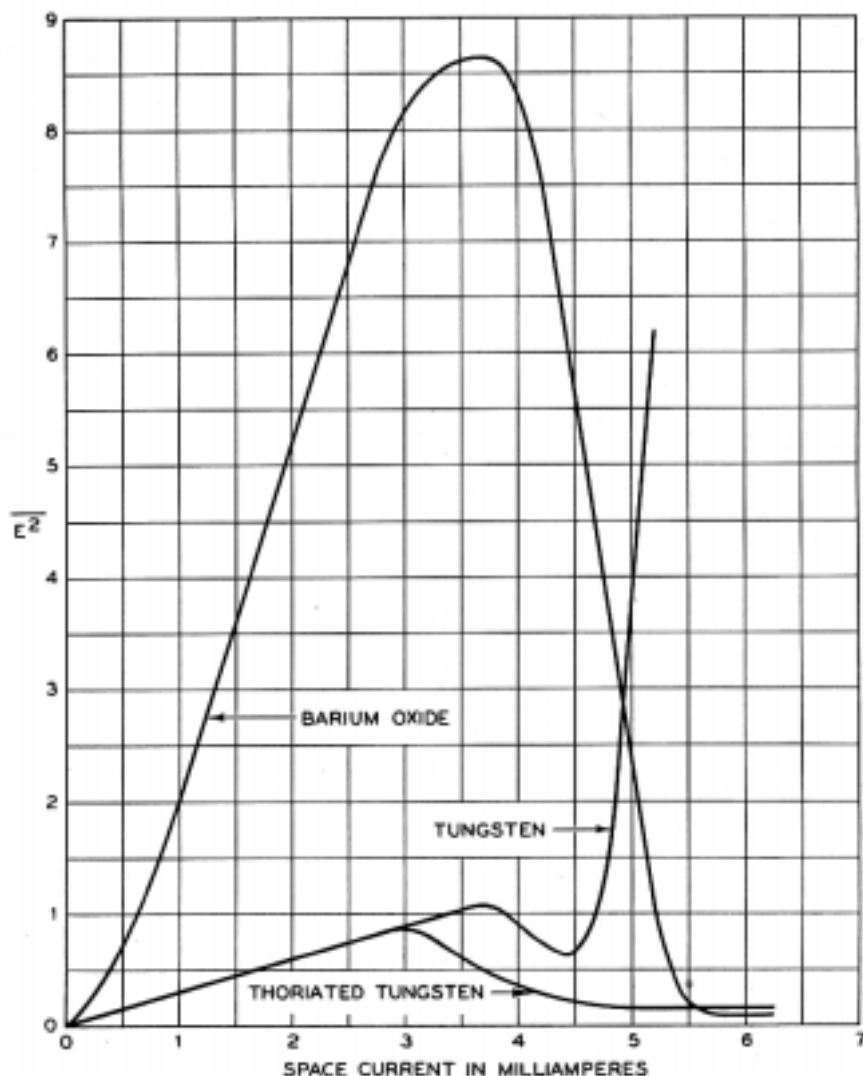

Fig. 1—The effect of space charge on fluctuation noise. Three tubes having filaments composed of tungsten, thoriated tungsten, and barium oxide. $\overline{E^2}$ is the mean square noise voltage across the output measuring device expressed in arbitrary units. The variation in space current was obtained by changing the cathode temperature, the plate voltage remaining constant.

milliamperes, and ordinates represent mean square noise voltage across the output measuring device expressed in arbitrary units. The change in space current was obtained by varying the filament heating current while the plate voltage remained constant. Tubes having thoriated tungsten, tungsten, and barium oxide cathodes were used.

At low space currents where no space charge is present the thoriated tungsten and tungsten filaments each give a pure shot effect, the mean square voltage increasing linearly with the space current. As the space current is increased further and space charge sets in, the shot voltage in each tube goes through a maximum and decreases with oncoming temperature saturation as suggested by equation (5). With the approach of complete temperature saturation the noise, however, does not decrease to zero in accordance with this equation. If it were possible to reach complete temperature saturation the residual noise would not be due to the shot effect, but rather to thermal noise in the plate circuit of the tube, positive ions and secondary emission within the tube, and other contributing causes. Usually this condition is approached in the better commercial tubes so that the contribution of true shot noise is a small part of the total noise.

If the methods used in obtaining equation (4) are applied to equation (5), it is found that the shot noise in the plate circuit of the tube produces the same effect in the output measuring device as a signal applied to the input circuit whose magnitude at the grid expressed in mean square volts is

$$\overline{V^2} = 2ej(\partial i/\partial j)^2 (r_p/\mu)^2 F. \tag{6}$$

This equation shows that the level of shot noise at the input is lowered by an increase in the cathode temperature, which increases the degree of temperature saturation, and by an increase in the ratio μ/r_p, which by definition is the transconductance of the tube, but is independent of the external load resistance. It should be remembered, however, that shot noise in the plate circuit should not fix the noise level in low noise vacuum tubes and that never, as is sometimes done in the literature, can the noise of an amplifier be calculated as pure shot noise in the plate circuit, for in the absence of space charge the tube would not be an amplifier.

Although space charge can counteract the effect of random electron emission from the cathode so that shot noise is reduced, other factors can alter the flow of current in such a way that the noise is increased. This is the case when changes in emission occur over small areas of the cathode, giving rise to an additional fluctuation which has been termed flicker effect.[7] This type of noise is particularly noticeable

[7] J. B. Johnson, *Phys. Rev.*, **26**, 71 (1925); W. Schottky, *Phys. Rev.*, **28**, 74 (1926).

with oxide coated cathodes. Since the flicker effect is due to localized variations in the emission of the cathode, one would expect it to disappear in the presence of a complete space charge condition.

The experimental curve for the barium oxide coated filament, Fig. 1, shows a flicker effect many times larger than the shot effect on which it is superimposed. At low space currents the mean square flicker effect voltage increases faster than the pure shot noise, a square law rather than a linear relationship being followed. As space charge sets in, the flicker effect voltage goes through a maximum and then decreases with increased space current in the same manner as does the shot effect voltage. In spite of the large flicker effect, as complete temperature saturation is approached the total noise is even less than that found with the thoriated tungsten filament which has no flicker effect. This illustrates clearly the effectiveness of space charge in smoothing the space current.

When the control grid of a vacuum tube is floating at its equilibrium potential, the noise level is much higher than when the grid is connected through an input circuit to the cathode. This increase in noise is primarily due to thermal noise in the extremely high input resistance of the tube and to shot noise arising from small grid currents.[8] The magnitude of the thermal noise may be calculated, knowing that the input impedance of the tube consists of its input resistance, r_g, in parallel with its dynamic grid-to-ground capacitance. In such a combination the real resistance component, $R(f)$, is related to the pure resistance, r_g, and the dynamic capacitance, c, according to the equation

$$R(f) = r_g/(1 + 4\pi^2 c^2 r_g^2 f^2).\tag{7}$$

According to equation (1) the mean square thermal noise input voltage is then

$$\overline{V_T^2} = 4kTr_g \int_F df/(1 + 4\pi^2 c^2 r_g^2 f^2).\tag{8}$$

With the grid floating at its equilibrium position (usually slightly negative with respect to the cathode) the grid current is composed of two components equal in magnitude but opposite in sign. The one component consists of electrons reaching the grid, while the other consists of positive ions reaching and electrons leaving the grid. The electrons are liberated from the grid by secondary emission, the photoelectric effect, thermionic emission, and soft X-rays. It should be pointed out that space charge does not reduce the noise produced by

[8] L. R. Hafstad, *Phys. Rev.*, **44**, 201 (1933).

the shot effect in any of these currents. The general shot effect equations [9] show that the magnitude of shot noise from these grid currents is

$$\overline{V_g^2} = 2ei_g r_g^2 \int_F df/(1 + 4\pi^2 c^2 r_g^2 f^2)^2, \tag{9}$$

where i_g is the sum of the grid currents regardless of sign.

Noise Produced by Secondary Effects

In this classification are grouped several sources of disturbance whose individual effects are very difficult to calculate and measure under the operating conditions of the vacuum tube. For this reason the following discussion will include only a general consideration of the more obvious contributing causes.

Although the cathode is the principal source of electrons which reach the plate, in actual practice electrons are produced by ionization of the gas molecules within the tube or by secondary emission resulting from bombardment of the tube elements. Electrons produced in this manner are drawn to the plate and generate noise which is not much affected by the space charge. Assuming a reasonable magnitude for the current produced in this manner it can be shown by the shot equations that noise from this source is usually negligible. In cases where the gas pressure within a tube is above normal, or in screen-grid and multi-grid tubes having high plate resistances and considerable secondary emission, the shot noise from secondary and ionization electrons may be of the same order of magnitude as thermal noise in the plate circuit.

Positive ions formed from ionized gas molecules or emitted from the tube elements are much more effective in producing noise since, instead of being drawn off to the plate, they are attracted into the space charge region where small disturbances in equilibrium produce large momentary fluctuations in space current. Due to their large mass the motions of the ions are relatively slow, so that they are very effective in this respect. This type of noise is quite disturbing in amplifying tubes for it tends to become a maximum at complete temperature saturation. This is illustrated very clearly in the noise measurements on the tungsten filament shown in Fig. 1. Here positive ions from the filament begin to show their effect as space charge sets in, the number of ions and the amount of noise increasing as temperature saturation is approached. As heard in the loud speaker, this noise consists of sharp crackling sounds which can easily be dis-

[9] E.g. Ref. 4 or 5.

tinguished from the steady rustling noise of the shot and thermal effects.

Ballantine [10] has recently made calculations and measurements on the noise due to positive ions from collision ionization in which he has shown that the mean square noise voltage is roughly proportional to the gas pressure within the tube and to the 3/2 power of the plate current. Comparing his results with equation (2), it appears that under ordinary working conditions the noise due to collision ionization in a vacuum tube may be of the same order of magnitude as noise from thermal agitation in its plate circuit. The noise level of tubes having a poor vacuum, however, may be much higher.

MEASUREMENT OF TUBE NOISE

The performance, as regards freedom from noise, of a vacuum tube used in an amplifier may be indicated by a comparison between the noise and a signal applied to the grid. Usually we say that the noise is equivalent to a signal which gives the same power dissipation in the output measuring instrument as the noise, the frequency of the signal being suitably chosen with respect to the frequency characteristics of the amplifier. Since tube noise is distributed over all frequencies and the noise power increases with the effective band width, it will be advantageous to express this input signal in equivalent mean square volts per unit frequency band width, effective over a given frequency range.

From these considerations it can be seen that the most convenient standard signal for measuring the equivalent input noise over any given frequency range is one in which the mean square signal voltage is distributed equally over all frequencies. With such a signal the equivalent input noise over any frequency range can be measured directly, while if an oscillator is used a number of measurements are required and the result must be computed by graphical integration. A signal which meets these frequency requirements perfectly is the noise of thermal agitation. Accordingly, in the measurements to be described here the standard input signal will be the thermal agitation voltage of a resistance R, connected between the control grid and cathode of the tube under test.[1]

The thermal noise voltage of the grid circuit, referred to the output measuring device, is given by equation (1), where $R(f)$ is the real resistance component of an input impedance consisting of the pure resistance R in parallel with its shunt capacity and that of its leads

[10] Stuart Ballantine, *Physics*, **4**, 294 (1933).

and of the vacuum tube. In such a combination $R(f)$ is related to the pure resistance R and the total capacitance c according to equation (7). In all the measurements described here the factor $4\pi^2 c^2 R^2 f^2$ is so small in comparison with unity that it may be neglected without appreciable error. Under these conditions equation (1) reduces to

$$\overline{E_T^2} = 4kTR \int_F |G_1(f)|^2 df, \tag{10}$$

where R is the direct current value of the resistance between control grid and cathode of the tube under test.

The voltage fluctuations arising from conditions within the tube produce a mean square voltage output $\overline{E_N^2}$ according to the equation

$$\overline{E_N^2} = \int_F |V(f)|^2 |G_1(f)|^2 df, \tag{11}$$

where $|V(f)|^2$ is the tube noise at the frequency f for unit frequency band width, expressed in volts squared and referred to the input circuit. Letting V_F^2 be the effective value of $|V(f)|^2$ over the band width of the amplifier we obtain

$$\overline{E_N^2} = V_F^2 \int_F |G_1(f)|^2 df. \tag{12}$$

Since the integrals in equations (10) and (12) are identical it is found on dividing one equation by the other and solving for V_F^2 that:

$$V_F^2 = 4kTR(\overline{E_N^2}/\overline{E_T^2}). \tag{13}$$

Equation (13) enables one to calculate the magnitude of tube noise in the frequency range F, per unit cycle band width, in terms of the thermal noise generated in a resistance R placed in the input circuit.[11] Since this equation contains no integral the measurements are simplified in that neither standard signal generator nor calibrated amplifier is required.

Apparatus

The experimental arrangement used in the measurements to be reported here is given in schematic form in Fig. 2. The system includes the tube under test, a high gain amplifier, appropriate filters, an attenuator, and an output measuring device.

[11] It is assumed that tube noise does not vary with frequency, or that the band width of the amplifier is so narrow that no appreciable error is introduced in applying the result.
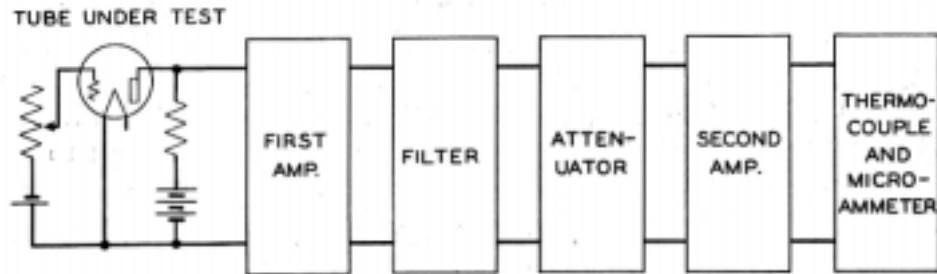
TUBE UNDER TEST

Fig. 2—Schematic amplifier circuit for measuring fluctuation noise in vacuum tubes.

The input circuit consists of the tube under test together with the variable grid resistor, external load resistor, and batteries for furnishing the required filament, grid, and plate voltages. Because of the high value of amplification required and the wide frequency range covered by the amplifier, this circuit required shielding from external disturbances arising from electrical, mechanical, and acoustical shock. Accordingly the tube under test was suspended by means of rubber bands, the whole circuit with the exception of batteries placed inside a tightly sealed lead lined box, and this box in turn suspended by means of a system of damped springs. The box with its cover removed and the tube in place is shown in Fig. 3. This shielding was sufficient to reduce the noise from outside disturbances to such a low level that no correction had to be made for it at any time.

The high gain amplifier [12] consists of two separate resistance coupled units each containing three stages. Each unit is so designed and shielded that the effect of external disturbances is eliminated. The total gain obtainable is about 165 db (constant to within 2 db from 10 cycles to 15,000 cycles). Since this gain is in excess of that required for the study of thermal and tube noises, an attenuator having a range of 63 db was inserted between the two units. In order to limit amplification to certain desired frequency bands, specially designed electric filters were inserted between the first amplifier unit and the attenuator. Three such filters were used of which one is a low-pass filter with cut-off around 205 cycles, and the other two are band-pass filters with mid-frequencies at 1750 and 11,000 cycles respectively. The frequency characteristic of the amplifier with no filter and with each filter inserted is shown in Fig. 4.

The recording instrument is a 600-ohm vacuum thermocouple and microammeter. Conveniently, the deflection of the microammeter is closely proportional to the mean square voltage applied to the couple. The procedure in making a measurement of tube noise is as follows:

[12] The essential parts of this amplifier were designed by Mr. E. T. Burton.
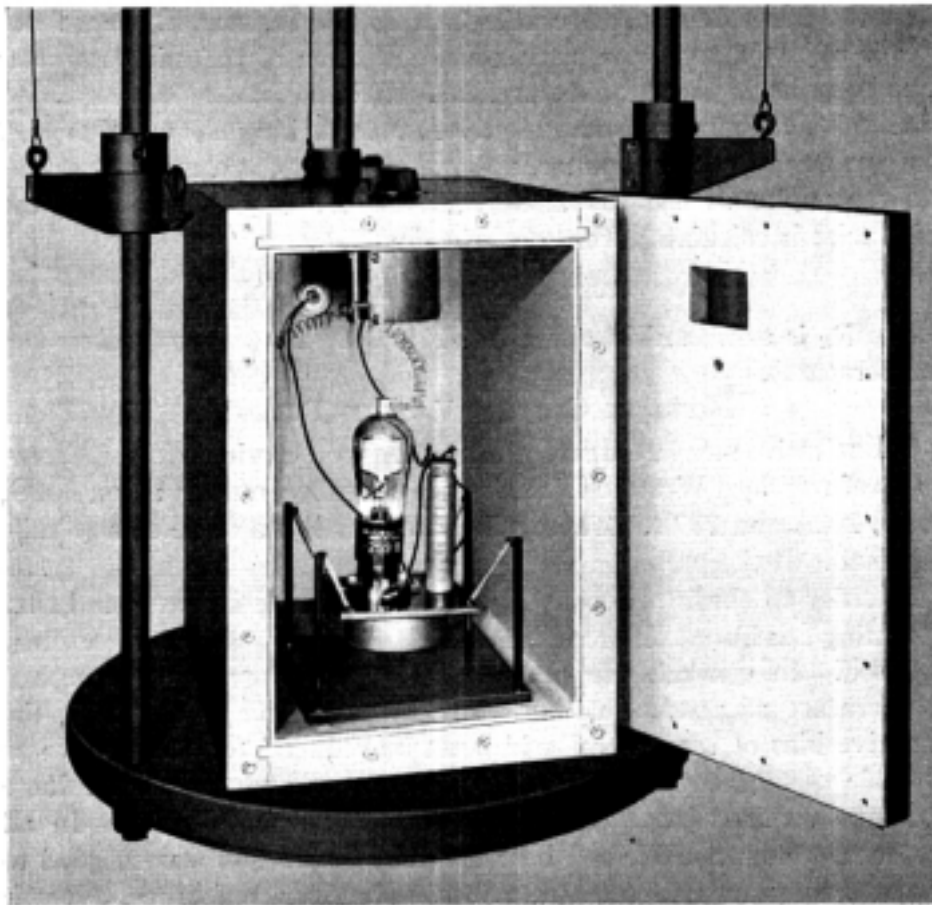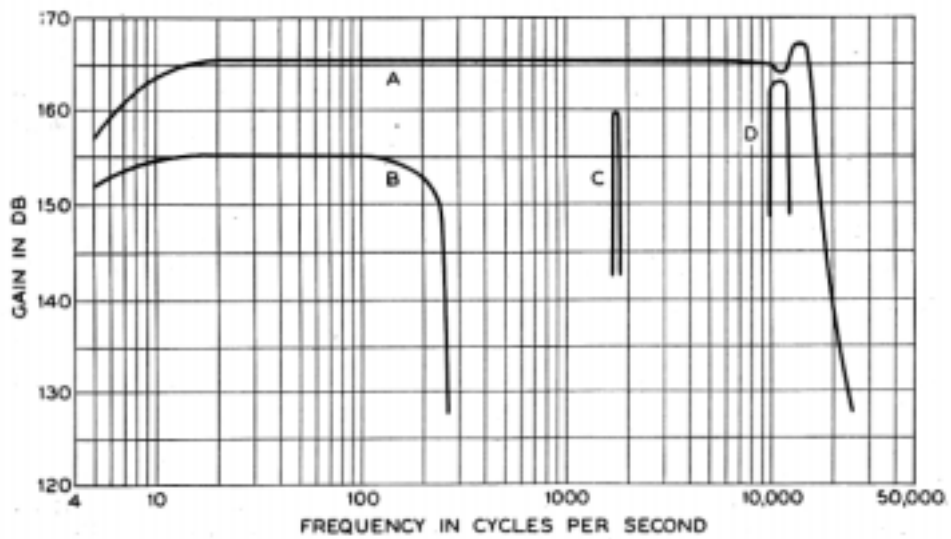
Fig. 3—Tube under test mounted in the shielding box.

Fig. 4—Frequency characteristic of amplifier circuit. Curve *A* with no filter, Curve *B* with low pass filter, and Curves *C* and *D* with band pass filters.

With the tube under test operating at zero grid resistance, the attenuator is adjusted to give a convenient deflection of the microammeter (due to noise in the tube under test). Grid resistance is now added until this deflection is exactly doubled, thus making $\overline{E_N^2}$ equal to $\overline{E_T^2}$. This value of input resistance, designated by R_G, is a measure of the inherent noise of the tube. Substituting R_G in equation (13) the tube noise is calculated from the relation

$$V_F^2 = 4kTR_G = 1.64 \times 10^{-20} R_G \text{ volt}^2, \qquad (14)$$

where R_G is expressed in ohms and T is 300° K. (approximate room temperature).

Noise in Certain Vacuum Tubes [13]

Quantitative measurements of tube noise were made on four different types of standard Western Electric vacuum tubes, namely: Nos. 102G, 264B, 262A and 259B. These tubes have as low a noise as any tube obtainable at the present time.

In order to obtain the best signal to noise ratios it was found that operating conditions different from those normally recommended must be used. In general, the cathode must be operated at as high a temperature as possible without impairing the life of the tube, the negative bias of the control grid must be reduced to as near zero as possible without causing excessive grid current, and the plate voltage must be reduced below the value normally recommended. In all the measurements described here the tube under test was coupled to the first amplifier unit through a 50,000-ohm load resistance. It was found that the signal-to-noise ratio could be improved a fraction of a db by increasing the load resistance (in accordance with equation (4)); this, however, necessitated a large plate voltage which was inconvenient. Six tubes of each type were tested and the noise data given below were obtained by averaging the six measurements for each type. Individual tubes may differ from these average values by as much as ± 1 db.

No. 102G Tube

This is a three-element, filament-type tube. Its long life, exceptionally high stability of operation, and good temperature saturation make it a desirable tube to use in the input stage of certain high-gain amplifiers. This tube also has a comparatively small microphonic response to mechanical and acoustical shock although it is not as good as the No. 262A and the No. 264B tubes in this respect.[14]

[13] Noise in other types of vacuum tubes has been reported by G. F. Metcalf and T. M. Dickinson, *Physics*, 3, 11 (1932); E. A. Johnson and C. Neitzert, *Rev. Sc. Inst.*, 5, 196 (1934); E. B. Moullin and H. D. M. Ellis, *I. E. E. Jour.*, 74, 323 (1934); W. Brentzinger and H. Viehmann, *Arch. f. Hochfr. und Elektroauk*, 39, 199 (1932).

[14] The microphonic response of several types of Western Electric vacuum tubes to mechanical agitation is reported by D. B. Penick in this issue of the *Bell Sys. Tech. Jour.*

The conditions found most suitable for quiet operation of the No. 102G tube and the corresponding average tube characteristics are given in the first two columns of Table I.   Under these conditions the

TABLE I

WESTERN ELECTRIC No. 102G TUBE

Operating Conditions	Tube Characteristics	Noise Data		
		Frequency Range Cycles per Sec.	V_F^2 Volt^2	R_G Ohms

Filament
  Voltage, 2.0 volts
  Current, 1.0 ampere
Grid
  Voltage, − 0.5 volt
Plate
  Voltage, 130 volts
  Current, 1.2 milli-
    amperes
Load
  Resistance, 50,000
    ohms
	Type of Tube, 3 Ele-
	  ment
	Type of Cathode, Ox-
	  ide Coated Fila-
	  ment............
	Amplification Factor,
	  30...............
	Plate Resistance,
	  45,000 ohms......
	Approx. Dynamic In-
	  put Capacitance,
	  80 μμf..........
		10–15,000
		
		5–205
		
		1,750–1,850
		
		10,000–12,000
			0.64×10^{-18}
			
			2.2
			
			0.58
			
			0.54
				3,900
				
				13,600
				
				3,550
				
				3,300

average equivalent tube noise voltage, referred to the grid circuit, is given in the last column of the same table.   These noise data are given in terms of R_G, the experimentally determined equivalent noise resistance of the tube, and in terms of V_F^2, calculated by means of equation (14), for each of the four frequency ranges shown in Fig. 4.

The No. 102G has the lowest noise of all the tubes tested and was found suitable for use in the first stage of high-gain amplifiers where tube noise is the limiting factor, provided it is not required that the input capacitance and microphonic response to mechanical and acoustical shock be extremely low.

No. 264B Tube

This is a three-element filament-type tube.   Due to the rigid construction and the short filament which is designed to reduce vibration to a minimum, the microphonic response of the tube to mechanical and acoustical shock is exceptionally low.[15]   The extensive system of spring suspensions and the heavy sound-proof chamber usually required for shielding low noise tubes may be simplified when using the No. 264B.   In addition, this tube has good temperature saturation, low power consumption, and high stability of operation.

The operating conditions and noise data for this tube are given in Table II.   Although the noise of this tube is slightly higher than that

[15] M. J. Kelly, *S. M. P. E. Jour.*, **18**, 761 (1932).

TABLE II

WESTERN ELECTRIC No. 264B TUBE

Operating Conditions	Tube Characteristics	Noise Data		
		Frequency Range Cycles per Sec.	V_p^2 Volt²	R_G Ohms

Filament
  Voltage, 1.5 volts
  Current, .30 ampere
Grid
  Voltage, − 0.5 volt
Plate
  Voltage, 26 volts
  Current, 0.6 milliampere
Load Resistance,
  50,000 ohms
	Type of Tube, 3 Element
	Type of Cathode, Oxide Coated Filament............
	Amplification Factor, 7................
	Plate Resistance, 18,500 ohms......
	Approx. Dynamic Input Capacitance, 30 μμf..........
		10–15,000
		
		5–205
		
		1,750–1,850
		
		10,000–12,000
			1.3 × 10⁻¹⁶
			
			6.6
			
			1.1
			
			1.0
				7,650
				
				40,000
				
				6,800
				
				6,200

of the No. 102G, the lower microphonic response and the lower power consumption make it a more desirable tube to use in input stages of certain high gain amplifiers.

No. 262A Tube

This is a three-element tube having an indirectly heated cathode. It is designed to give a microphonic response to mechanical and acoustical shock [15] still lower than that of the 264B. Except for frequencies below 200 cycles per second it was found that no acoustic shield was necessary for this tube even when working at extremely low levels. Although this tube is designed to have a low hum disturbance resulting from alternating current for heating the cathode (the interference from this effect can be held to less than 7×10^{-6} equivalent input volt), direct current power was used in the measurements here described.

The operating conditions and noise data for the No. 262A tube are given in Table III.

No. 259B Tube

This is a four-element, screen-grid tube having an indirectly heated cathode. Its comparatively high amplification factor makes possible a relatively large gain per stage so that when it is used in the first stage of a high-gain amplifier succeeding stages contribute nothing to the total noise.

Noise measurements on the No. 259B tube show that the signal-to-noise ratio is approximately independent of the plate voltage over a

TABLE III

WESTERN ELECTRIC No. 262A TUBE

Operating Conditions	Tube Characteristics	Noise Data		
		Frequency Range Cycles per Sec.	$V p^2$ Volt²	R_G Ohms

Heater
    Voltage, 10 volts
    Current, 0.32 ampere
Grid
    Voltage, − 1.0 volt
Plate
    Voltage, 44 volts
    Current, 1.0 milliampere
Load Resistance,
    50,000 ohms
	Type of Tube, 3 Element
	Type of Cathode, Oxide Coated, Indirectly Heated.....
	Amplification Factor, 15.7.............
	Plate Resistance, 22,000 ohms......
	Approx. Dynamic Input Capacitance, 23 μμf..........
		10–15,000
		
		5–205
		
		1,750–1,850
		
		10,000–12,000
			1.3 × 10⁻¹⁶
			
			17.
			
			1.0
			
			0.84
				7,700
				
				100,000
				
				6,400
				
				5,100

wide operating range, but is closely dependent on the plate current as affected by the control and screen grid voltages. Table IV contains the operating conditions and noise data for this tube.

Noise measurements were also made on the No. 259B tube with its control grid floating at equilibrium potential. Using the operating voltages specified above, the noise level was about 20 db higher than those given in Table IV. The level can be greatly reduced by operating the tube at a lower cathode temperature and with lower screen

TABLE IV

WESTERN ELECTRIC No. 259B TUBE

Operating Conditions	Tube Characteristics	Noise Data		
		Frequency Range Cycles per Sec.	$V p^2$ Volt²	R_G Ohms

Heater
    Voltage, 2.0 volts
    Current, 1.7 amperes
Grid
    Control Voltage, − 1.5 volts
    Screen Voltage, 22.5 volts
Plate
    Voltage, 100 volts
    Current, 0.6 milliampere
Load Resistance, 50,000 ohms
	Type of Tube, 4 Element Screen Grid
	Type of Cathode, Oxide Coated, Indirectly Heated.....
	Amplification Factor, 1,500.............
	Plate Resistance, 2.75 megohms.....
	Approx. Dynamic Input Capacitance, 6.0 μμf..........
		10–15,000
		
		5–205
		
		1,750–1,850
		
		10,000–12,000
			3.2 × 10⁻¹⁶
			
			7.7
			
			2.8
			
			2.8
				19,800
				
				47,000
				
				17,100
				
				17,000

and plate voltages.[16]   This reduction in noise is due to a decrease in current to the floating grid.   Using a heater current of 1.3 amperes, a plate current of 0.1 milliampere, a screen potential of 16.5 volts and a plate potential of 30 volts the equivalent input noise was 1.4×10^{-5} volt for the entire frequency range from 10 cycles to 15,000 cycles. Under these operating conditions the floating grid potential was 1.0 volt negative with respect to the cathode, the input resistance 1.4 $\times 10^{10}$ ohms, the dynamic grid-to-cathode capacitance 6×10^{-12} farad, and each component of grid current about 4.5×10^{-12} ampere.

DISCUSSION OF RESULTS

From the noise data in the preceding tables one can estimate quite accurately the equivalent input noise voltage of each of the four types of tubes at any frequency between 5 and 15,000 cycles, and for any band width within these limits.   For example, using the noise data given in Table I the equivalent input noise voltage of the No. 102G tube working over a band having sharp cut-offs at 5 cycles and 205 cycles is computed to be

$$(\overline{V^2})^{1/2} = (V_F{}^2 F)^{1/2} = 2.1 \times 10^{-7} \text{ volt.} \tag{15}$$

For a band width of 200 cycles with mid-frequency at 10,000 cycles this noise is reduced to 1.0×10^{-7} volt.   It can be seen that for each type of tube the noise voltage over equal band widths is between 1.5 and 4.5 times greater at frequencies below 200 cycles than at the higher frequencies.[17]

Even at high frequencies the noise voltage is above that expected from thermal noise in the plate circuit which, as stated above, is the absolute minimum to which fluctuation noise in a thermionic vacuum tube may be reduced after all other causes are eliminated.   In the case of the No. 102G tubes for instance, using the operating conditions of Table I, and assuming 1100° K. as the temperature of the barium oxide filament, it is found by means of equation (4) that the equivalent input noise voltage produced by thermal agitation in the plate circuit is 2.7×10^{-8} volt for a band width of 200 cycles.   The total input noise voltage obtained experimentally at the higher frequencies is greater than this by a factor of 3.8.   In like manner it is found that the total input noise voltages found experimentally for the Nos. 264B, 262A and 259B tubes are greater than the equivalent input thermal

[16] I am indebted to Dr. J. R. Dunning of Columbia University for pointing out this fact.

[17] Other investigators have also found an increase in tube noise energy at the lower frequencies.   G. F. Metcalf and T. M. Dickinson, *Physics*, **3**, 11 (1932).

noise voltages produced in the plate circuit by factors 2.1, 3.7, and 16 respectively.   These calculations show that each of these four types of tubes approaches the requirements of an ideal low noise amplifying tube although none of them is perfect in this respect.

As stated above, the best signal-to-noise ratio in a high-gain amplifier is obtained when thermal agitation in the input resistance is responsible for most of the noise in the amplifier.   This condition is met when the resistance of the input circuit is higher than the value of R_G for the input tube.   In case the resistance in the input circuit is less than R_G the input signal and the thermal noise from the input circuit can be raised above the noise of the tube by using an input transformer having a sufficiently high voltage step-up.   The voltage ratio of the transformer, and in turn the possible ratio of input circuit thermal noise to tube noise, is limited, especially at the higher frequencies, by the dynamic grid-to-ground capacitance of the input tube and its leads.   In such a circuit the No. 259B tube with its lower inter-electrode capacities and higher tube noise is often more desirable than even the quietest three-element tubes.

In those high-gain amplifiers in which unavoidably the resistance of the input circuit is low, the tube rather than thermal agitation in the grid circuit is responsible for most of the noise.   Here the best signal-to-noise ratio can be obtained by choosing a tube for the initial stage having the lowest possible noise level.   The above measurements show that one of the three-element tubes, particularly the No. 102G tube if sufficient shielding is used, is best suited for this purpose.

The lower limits of noise obtainable with high gain amplifiers may be estimated by means of Fig. 5, which shows the noise as a function of input resistance and frequency band width when thermal agitation in the input circuit is responsible for all the noise.   The data for this figure are obtained from the thermal noise relationship

$$\overline{V_T^2} = 1.64 \times 10^{-20} RF \text{ volt}^2. \tag{16}$$

R is expressed in ohms and the temperature has been taken at 300° K., which is approximately room temperature.   It must be remembered that the attainment of these noise levels at low input resistances is limited by the input transformer.

The results of the noise measurements on the No. 259B tube with floating grid may be compared with the value predicted by equations (8) and (9).   Inserting the tube characteristics obtained by experiment ($r_g = 1.4 \times 10^{10}$ ohms, $i_g = 9 \times 10^{-12}$ ampere, and $c = 6 \times 10^{-12}$ farad), and integrating between the frequency limits 10 cycles
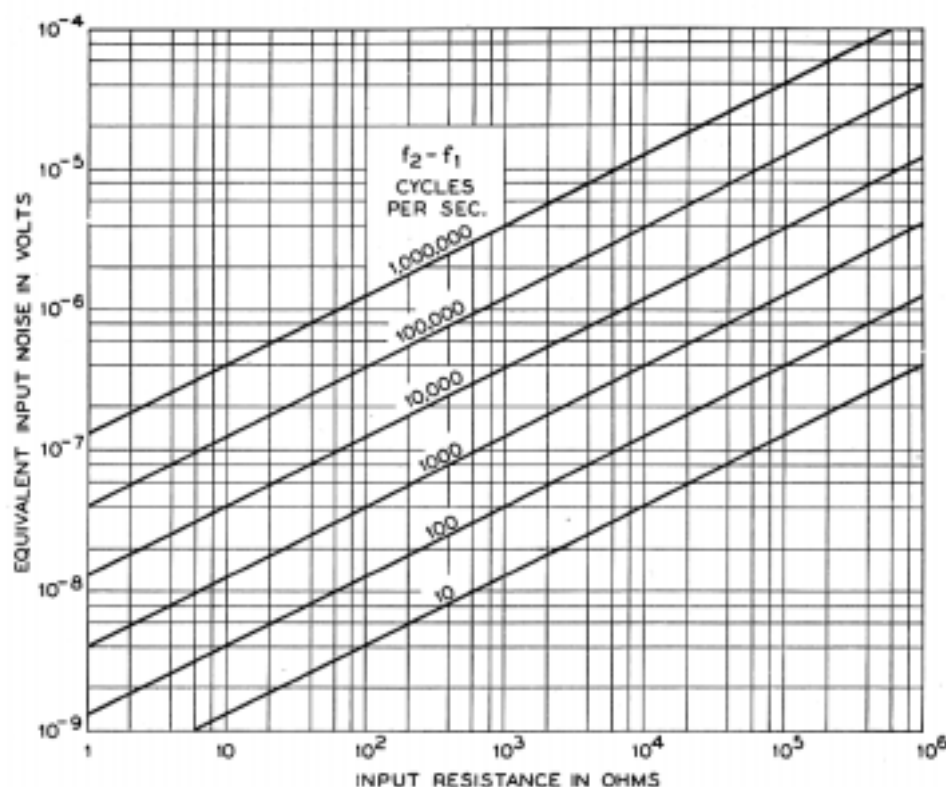
Fig. 5—Thermal noise level as a function of input resistance and frequency range.

and 15,000 cycles, it is found that the equivalent thermal noise input is 0.9×10^{-5} volt, while the shot noise input is 1.4×10^{-5} volt. The total noise is the square root of the sum of the squares of these values or 1.7×10^{-5} volt. This agrees with the measured value of 1.4×10^{-5} volt within an error of 20 per cent, which is as accurate as the determination of the grid currents. These equations may also be used to calculate the noise originating in the grid circuit when external resistance or capacitance is connected between grid and cathode, provided r_g and c are now calculated from the internal and external impedances in parallel.

A common method of detecting corpuscular or electromagnetic radiation makes use of an ionization chamber and linear amplifier. In this circuit the control grid in the first tube of the amplifier is connected to the collecting electrode of the ionization chamber and both allowed to float at equilibrium potential.[18] The shot and thermal noise in this grid circuit sets a limit to the measurement of extremely weak radiation. Knowing the value of input capacitance, input

[18] H. Greinacher, *Zeits. f. Physik*, **36**, 364 (1926).

resistance, floating grid current, and the frequency limits of the amplifier, equations (8) and (9) may be used to calculate this limiting noise level. For example, if one uses a No. 259B tube with the operating voltages specified for floating grid, an ionization chamber having a capacitance of 15×10^{-12} farad, and an amplifier having a frequency range from 200 to 5000 cycles per second the limiting noise level is 1×10^{-6} root mean square volt.

The limiting noise level in a system consisting of a photoelectric cell and thermionic amplifier is determined by thermal agitation in the coupling circuit between the photoelectric cell and amplifier, and by shot noise in the photoelectric current (in circuits where the photoelectric current is very small and the coupling resistance is very high, shot noise from grid current in the vacuum tube becomes appreciable). The noise of thermal agitation may be calculated by means of equation (8) provided r_g is now replaced by R, the coupling resistance. If vacuum cells are used, the photoelectric current produces a pure shot noise which can be calculated by equation (9) provided i_g is replaced by I, the photoelectric current. In gas filled photocells where collision ionization occurs, the noise is in excess of the value calculated in this manner.[19] The relative magnitude of shot noise and thermal noise depends on the values of I and R, and by combining equations (8) and (9) it is found that

$$\overline{V_s^2}/\overline{V_T^2} = eIR/2kT = 19.4IR, \tag{17}$$

where I is expressed in amperes, R in ohms, and T is $300°$ K. Thus an increase in either I or R will tend to make shot noise exceed thermal noise. This is the desirable condition since it furnishes the largest ratio of signal-to-noise for a given light signal on the photoelectric cell.

In conclusion I wish to acknowledge my indebtedness to Dr. J. B. Johnson for the helpful criticism he has given during the course of this work.

[19] B. A. Kingsbury, *Phys. Rev.*, **38**, 1458 (1931).